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Tunable shape-shifting structures for military applications

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14. ABSTRACT

Surfaces that modify their texture (potentially multiscale) in response to either direct control or automatic feedback from external cues (solar light intensity, temperature, incoming waves, etc.) are particularly attractive for military applications as they could be used to change radar, acoustic, thermal or optical signatures to fulfill mission-specific objectives. A natural question would be: how does one design a surface that can dynamically change shape and have its texture smoothly and reversibly reconfigured into specified complex three-dimensional patterns at specific spatial scales? How to control the deformation of active elements made of metamaterials to obtain these textured patterns? The goal of this project was to provide a proof-of-concept answer to this question and, by doing so, effectively bridging the gap between the integration of metamaterials and the design of reconfigurable active structures. A robust computational modeling platform was developed based on non-linear finite element and optimization procedures to design and optimize patterned surfaces by using a clever combination of materials, structural layers, boundary and loading conditions (active controls). The idea was to control three-dimensional surfaces by exploiting their local buckling/wrinkling/folding characteristics triggered by smooth deformation of active reinforcement elements. The proof-of-concept approach was successfully demonstrated. It is possible to deform the surface of a multilayer soft structure with a high level of control to match a given target three-dimensional surface topography by inducing controlled localized buckling. The multiphysics computational platform that was developed is general and can accommodate various types of active materials acting as controllable actuators.

15. SUBJECT TERMS

EOARD, camouflage, biomimetics, active materials, adaptive, multi-scale structural mechanics, metamaterials, finite elements

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Executive summary

Background

Metamaterials and smart active structures offer a huge potential to develop multifunctional systems addressing the technological needs of army forces. Like evolutionary-optimised biological systems found in Nature (e.g. adaptive skin texture of cephalopods), active soft structures producing large deformations offer attractive ways to incorporate the ability to reconfigure their surface texture. Surfaces that modify their (potentially multiscale) texture in response to either direct control or automatic feedback from external cues (solar light intensity, temperature, incoming waves, etc) are particularly attractive as they could be used to change radar, acoustic, thermal or optical signatures to fulfil mission-specific objectives.

Objectives

A natural question would be: how does one design a surface that can dynamically change shape and have its texture smoothly and reversibly reconfigured into specified complex three-dimensional patterns at specific spatial scales? How to control the deformation of active elements made of metamaterials to obtain these textured patterns?

The goal of this project was to provide a proof-of-concept answer to this question and, by doing so, effectively bridging the gap between the integration of metamaterials and the design of reconfigurable active structures.

Approach

Here, it was proposed to develop a robust computational modelling platform based on non-linear finite element and optimisation procedures to design and optimise patterned surfaces by using a clever combination of materials, structural layers, boundary and loading conditions (active controls). The idea was to control three-dimensional surfaces by exploiting their local buckling/wrinkling/folding characteristics triggered by smooth deformation of active reinforcement elements.

Results

The proof-of-concept approach was successfully demonstrated. It is possible to deform the surface of a multi-layer soft structure with a high level of control to match a given target three-dimensional surface topography by inducing *controlled localised* buckling. The multiphysics computational platform that was developed is general and can accommodate various types of active materials acting as controllable actuators.

Keywords : Camouflage, Biomimetics, Active, Adaptive, Morphing, Surface, Structure, Multi-Scale Structural Mechanics, Metamaterials, Theoretical and Computational Methods, Finite Element, Isogeometric, NURBS, Optimisation

1. Introduction

1.1 Background

At the micro- and nanoscale, the periodicity of patterned surfaces present very interesting properties such as unique microfluidic [1], optical, electronic or acoustic properties because of wave interference and/or transformation [2]. In Nature, a number of animals and plants have evolved the ability to exploit patterned surfaces at various scales (e.g. Lotus flower, shark and its skin cuticles) to their advantages [3]. *Dynamic* or *adaptive* patterned surfaces are very desirable as they can be adapted to specific contexts, operating conditions and environments. Cephalopods such as octopus (**Figure 1**), squid and cuttlefish possess some of the most striking abilities in the animal kingdom: camouflage via colour change and adaptive change of the three-dimensional texture of their skin [4-6]. In a military context, these characteristics are very attractive and can open up a vast array of applications.

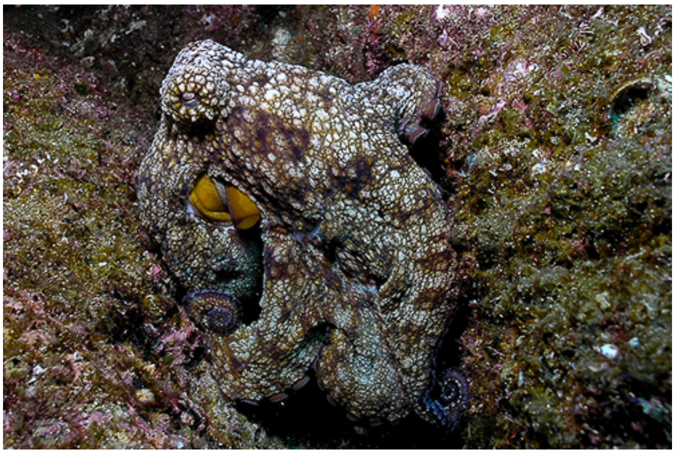


Figure 1. Octopus demonstrating its camouflage abilities in its natural environment (Sea of Cortez, Mexico, G. Limbert).

1.2 Military relevance

Metamaterials and smart active structures are particularly relevant to the technology requirements of army forces. The ever growing trend is to develop autonomous (also self-healing and fault-tolerant) systems that can provide multifunctionality whilst minimising weight/size, particularly in extreme environments (underwater, air and space). The solution to these challenges is to develop single systems that can be reconfigured for specific missions / applications. Like evolutionary-optimised biological systems found in Nature (e.g. cephalopods), active soft structures producing large deformations offer attractive ways to incorporate the ability to reconfigure their surface texture. Surfaces that modify their texture in response to either direct (human/computer-controlled) orders or automatic feedback from external cues (solar light intensity, temperature, incoming waves, etc) would provide the army forces with a distinct technological advantage in a wide variety of scenarios and applications. This would make such multifunctional active structures particularly attractive as they could be used to change radar, acoustic, thermal or optical signatures to fulfil specific mission requirements.

> *A natural question would be how does one design a surface that can dynamically change shape and have its texture smoothly and reversibly reconfigured into specified complex three-dimensional patterns at specific spatial scales? How to control the deformation of active elements made of metamaterials to obtain these textured patterns?*

The goal of this project was to provide a proof-of-concept answer to this question.

Here, it was proposed to **develop a robust computational modelling platform based on non-linear finite element and optimisation procedures to design and optimise patterned surfaces** by using a clever combination of materials, structural layers, boundary and loading conditions (active controls). The idea was to control three-dimensional surfaces by exploiting their local buckling/wrinkling/folding characteristics triggered by smooth deformation of active reinforcement elements. Dynamic reconfiguration of the texture could be done by using electroactive [7] or magnetoactive materials [8].

The proposed proof-of-concept innovation could be used to assist engineers in DOD-related industries and laboratories to accelerate the understanding, design and development of smart active surfaces by exploiting, *in a robust and systematic way*, the physics of metamaterials [e.g. electroactive polymers (EAPs)] to propose innovative solutions for military forces. *This would effectively close the gap between the integration of metamaterials and the design of reconfigurable active structures*. The main unique selling point of the concept proposed in this project is to provide and integrated computational modelling environment so that novel cost-effective solutions could be explored in a minimum amount of time (computational optimisation studies).

1.3 Proposed approach

The proposed computational platform is based on non-linear finite element and numerical optimisation techniques.

Because of the novelty and highly non-linear nature of the problems to be simulated (see **Remark 1**), new and/or improved finite element techniques and numerical algorithms had to be developed for this project (they are detailed in section **2**). To efficiently capture wrinkling instabilities, it was proposed to develop a robust isogeometric (NURBS-based) finite element framework [9] (**work package 1**, section **2.2**) and an improved arc-length non-linear finite element solver (**work package 2**, section **2.3**). A special optimisation procedure had to be developed to establish local active deformation patterns of the top surface of a multi-layer structure required to produce a final target shape (**work package 3**, section **2.4**). For subsequent exploitations of the computational framework post-processing and visualisations algorithms were also developed (**work package 2**, section **2.5**).

Finally, the computational platform was exploited and proof-of-concept results are provided in section **3**.

Remark 1

Mechanics of wrinkle formation: *the theoretical study of wrinkling has recently received much attention in the physics and soft matter communities [10-13] as they are relevant for a wide range of industrial applications and fundamental problems. Wrinkles form in response to competing physical constraints such as inextensibility, mismatch of stiffness in multi-layer structures and specific multi-axial loading conditions together with a principle of energy minimisation [12]. Wrinkling of initially flat (linearly) elastic thin sheets and their non-linear kinematics are governed by the fourth-order non-linear Föppl-von Kármán equations [14] which cannot be solved analytically in most cases, particularly if one considers complex geometry, boundary and loading conditions. The bending energy term, critical to wrinkling, is proportional to the second derivative of the local curvature of the shell mid-surface and, in a finite element context, this means that* C^1*-continuity across finite element boundaries is required [15, 16].*

Remark 2

In the original project proposal, it was proposed to conduct a high-throughput computer experiment to assess the characteristics of wrinkles created by using a large number of combinations of material and structural parameters for a bi-layer structure as well as various uniform and non-uniform boundary and loading conditions. From this design of computer experiment, a surrogate model (also called meta-model) would have been built and used to calculate the sensitivities of the physical system to the design variables as well as cross correlations between these variables [17]. It was hypothesised that the exploitation of the surrogate model would provide fundamental insights into the relations existing between wrinkle patterns, wavelength, amplitude, mechanical and underlying structural properties. By analysing the direct sensitivities and output response of the surrogate model predictive laws would have been established. The surrogate model would have been subsequently used to conduct (computationally cheap) multiobjective numerical optimisation to seek specific wrinkle characteristics (and therefore specific three-dimensional texture) [17, 18].

This idea was later replaced by that of conducing direct optimisation studies to determine parameters necessary to obtained particular target surfaces (see sections 2.4 and 2.5).

2. Software development

For this project, extensive software development was conducted and consisted of the following main work packages:

- **WP1:** Development of isogeometric finite element formulations and their numerical implementation
- **WP2:** Development of inverse non-linear finite element optimisation procedures
- **WP3:** Development and implementation of a robust arc-length solver for wrinkling analysis
- **WP4:** Development and implementation of advanced algorithms for tensor field visualisation

The computational platform was exploited to address the objectives of the research project, namely demonstrating the feasibility of controlling the 3D texture of multi-layered soft structures by inducing localised buckling behaviour (see section **3**)

2.1 General software development platform for mathematical formulations and numerical implementations

In the initial phase of the project, the implementation of an isogeometric thin shell element and associated finite element procedures were carried out using the high-level language and numerical environment of **MATLAB®** (MathWorks, Inc., Natick, MA, USA) (see **Appendix A – Journal paper submitted**). The choice of this particular platform was mainly made because of the extensive experience of the post-doctoral researcher in using MATLAB®.
In the second phase of the project, a comparable environment based on **Mathematica®** (Wolfram Research, Inc., Champaign, IL, USA) was thoroughly used. This environment allowed us to significantly accelerate the development and this choice will prove very judicious indeed for any future iterations of the project. A symbolic-numeric Mathematica® plugin suite **AceGen/AceFEM** (http://www.fgg.uni-lj.si/Symech/) was used. The Mathematica® package AceGen is used for the automatic derivation of formulae needed in numerical procedures. Symbolic derivation of the characteristic quantities (e.g. gradients, tangent operators, sensitivity vectors, etc) generally leads to exponential growth of derived expressions, both in time and space. A very robust and efficient approach implemented in AceGen avoids this problem by combining several techniques: symbolic and algebraic capabilities of Mathematica®, automatic differentiation technique, automatic code generation, simultaneous optimisation of expressions and theorem proving by stochastic evaluation of the mathematical expressions [19]. The multi-language capabilities of AceGen can be used for a rapid prototyping of numerical procedures in script languages of general problem solving environments like Mathematica or MATLAB® as well as to generate highly optimised and efficient compiled language codes in Fortran or C. Through a unique user interface the derived formulae can be explored and analysed.
The AceGen package also provides a collection of prearranged modules for the automatic creation of the interface between the automatically generated code and the numerical environment where the code would be executed. The AceGen package directly supports several numerical environments such as: MathLink connection to Mathematica®, AceFEM which is a research finite element environment based on Mathematica® or research and commercial finite element packages based on the Fortran language. The multi-language and multi-environment capabilities of the AceGen package enable the **generation of numerical codes for various numerical environments from the same symbolic description**. In combination with the finite element environment AceFEM the AceGen package represents an ideal tool for rapid development of new numerical formulations and algorithms. The visualisation platform that was used was ParaView (www.paraview.org).

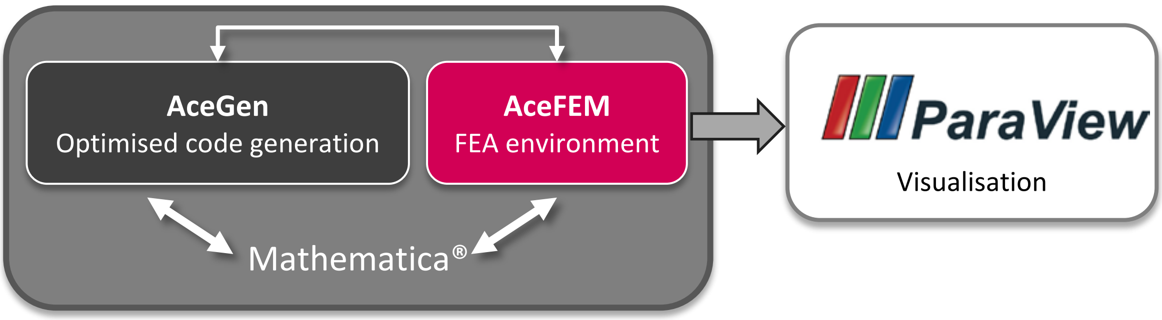


Figure 2. Software environment used.

2.2 WP1: Development of isogeometric finite element formulations and implementation

This work package consisted in implementing a generic modular framework for 2D and 3D isogeometric finite element formulations [9] into the environment of AceFEM (see section **2.1**). As mentioned in the **Introduction** section, mechanical instabilities involving wrinkling and folding feature high curvature gradients that can only be fundamentally captured by using C^1 formulations [16] and these imply continuity of the displacement field across boundaries of finite elements. Most "traditional" Lagrange polynomial-based finite element formulations do not satisfy these requirements unlike elements based on B-Spline and/or non-uniform rational B-Splines (NURBS) [9, 20]. The idea of using NURBS [21] as basis functions for analysis was introduced by Hughes et al. [9, 20], and was named *isogeometric analysis*. In isogeometric analysis, the functions for the geometry description are also used as basis functions for the analysis. Thus, the analysis works on a geometrically exact model. *This offers the possibility to close the existing gap between design and analysis by merging design geometry and analysis model*. It was demonstrated that not only were NURBS applicable to engineering analysis, but that they were better suited for many applications, and were able to deliver accuracy superior to standard finite elements (see, e.g., [22-30]). More importantly, NURBS are smooth, higher order functions which have become standard in CAD (computer aided design) programs. They allow great geometric flexibility and high order continuities at the same time. NURBS are therefore ideally suited as basis functions for Kirchhoff-Love shell [15, 31-33].
To be compatible with traditional data structures used in finite element software applications, the architecture of the isogeometric numerical implementation was based on the use of a Bézier extraction operator [34] which basically maps NURBS to Bézier polynomial bases and therefore offers a "traditional" finite element data structure for elements.

The developed isogeometric framework works for NURBS-based geometries defined by tensor products up to cubic polynomial order. The framework, which is a collection of specially written Mathematica® functions and C codes, can handle 2D and 3D multiphysics (i.e. multi-fields) element formulations and can accommodate material constitutive laws of arbitrary complexity. Natural, essential and initial boundary conditions are seamlessly integrated without the need to use penalty functions to enforce them. Edge and surface loads are also accounted for. By making use of the powerful differentiation capabilities of AceGen, all algorithms and element formulations are *consistently* linearised so that non-linear solution procedures converge *quadratically* [35].

Dedicated direct sensitivity subroutines were also developed. In a finite element context, the object of a sensitivity analysis is to calculate the dependence of a functional response $\mathfrak{R}(\mathbf{q}) = \{\mathscr{R}_1(\mathbf{q}), \mathscr{R}_2(\mathbf{q}), \ldots, \mathscr{R}_s(\mathbf{q})\}$ made of s scalars on n parameters $\mathbf{q} = \{q_1, q_2, \ldots, q_n\}$ which are generally arbitrary analysis model inputs (constitutive parameters, geometric characteristics of the finite element mesh, boundary and loading conditions) but can also be arbitrary intermediate or final results of the analysis. This is achieved by calculating the local partial derivatives of the functional response with respect to the parameters leading to a sensitivity matrix $\mathfrak{S}$:

$$\mathfrak{S} = \frac{\partial \mathfrak{R}(\mathbf{q})}{\partial \mathbf{q}} \quad [1]$$

The physical interpretation of a calculated sensitivity is that it provides the quantitative change of the response variable as result of a unit change in the parameter considered. Sensitivities can therefore be exploited in numerical optimisation studies and accelerate convergence of the procedure (this approach is followed in WP2, section **2.4**). In the context of this project numerical subroutines were developed to calculate three types of sensitivity:

- **Primal sensitivity analysis:** sensitivity of the primal field variables (3 displacement degrees of freedom for each node of the mesh (u, v, w)) with respect to the constitutive parameters.
- **Shape sensitivity analysis:** sensitivity of the primal field variables (3 displacement degrees of freedom for each node of the mesh (u, v, w)) with respect to the geometrical characteristics of the mesh (and the location of the boundary conditions).
- **Secondary sensitivity analysis:** sensitivity of selected response variables (e.g. Cauchy stress tensor [σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{xz} and σ_{yz}] and von Mises stress) with respect to the constitutive parameters.

In this work package, post-processing Mathematica® functions were also developed to project NURBS element results to linear 2D and 3D elements so that results of isogeometric analyses could be visualised in ParaView (see section **2.5**). A dedicated mean-square fitting of Gauss-points fields with user-controlled mesh refinement was devised.

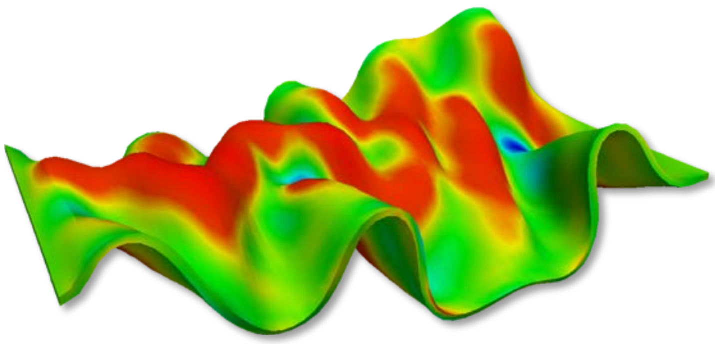


Figure 3. Deformed shape of a thin structure after a highly non-linear isogeometric analysis post-buckling procedure. Colour plot corresponds to the first principal Green-Lagrange strains projected from the NURBS elements of the isogeometric analysis to a mesh consisting of tri-linear hexahedron elements.

Deliverables

- NURBS-based Isogeometric finite element framework for primal and direct sensitivity analyses, and post-processing for static and dynamic non-linear solving procedures based on implicit scheme [35].
- 2D and 3D continuum elements implemented (linear, quadratic and cubic polynomial order) with possibility to couple kinematically 2D and 3D elements.
- Structural element: Mindlin-Reissner general shell which degenerates to a Kirchhoff-Love thin to ultra-thin shell in the limit of vanishing shell thickness without the typical associated pathological locking.
- Periodic boundary conditions implemented.

2.3 WP3: Development and implementation of a robust arc-length solver for wrinkling analysis

Post-buckling behaviour of structures (e.g. wrinkling) is a typical type of non-linear behaviour where a standard Newton-Raphson procedure can fail. Arc-length solution methods offer a good alternative [36, 37]. An arc-length iterative solution procedure solves consistently a linearised augmented system which is composed of equilibrium equations:

$$\mathbf{R}(\Delta\mathbf{p}, \Delta\lambda) = \mathbf{0} \tag{2}$$

and constraint equation:

$$g(\Delta\mathbf{p}, \Delta\lambda) = 0 \tag{3}$$

for a set of variables $\mathbf{p}$ and λ with unknown values which respectively correspond to the vector containing the global nodal degrees of freedom and load multiplier. The constraint equation is defined as:

$$g(\Delta\mathbf{p}, \Delta\lambda) = \Delta\mathbf{p}^{\mathrm{T}}\Delta\mathbf{p} + \Delta\lambda^2\Psi_1^2\mathbf{R}_{ref}^{\mathrm{T}}\mathbf{R}_{ref} + \Delta\lambda^2\Psi_2^2\mathbf{p}_{ref}^{\mathrm{T}}\mathbf{p}_{ref} - \Delta s = 0 \tag{4}$$

and considers an increment of arc-length of the equilibrium path Δs to determine the incremental variables $\Delta\mathbf{p}$ and $\Delta\lambda$.
Ψ_1 and Ψ_2 are scaling parameters while $\mathbf{R}_{ref}$ and $\mathbf{p}_{ref}$ are respectively the reference load vector (natural boundary conditions) and the reference vector of prescribed displacements (essential boundary conditions).

Generally, arc-length methods are based on a two-step solution strategy. The first step is a *predictor* phase which executes the first iteration and defines the direction of the path following procedure (positive or negative) in the current increment. For this reason, a judicious criterion to determine the correct direction is required. The second step is a *corrector* phase and executes the subsequent iterations in the current increment.
The implementation of the arc-length solver into AceFEM was based on the work of de Souza Neto and Feng [38] for an efficient criterion to determine the path direction and followed a similar approach to that of Schweizerhof and Wriggers [39]. Consistent linearisation of the solution procedure ensures quadratic rate of convergence of the Newton-type algorithm.

2.4 WP2: Development of inverse non-linear finite element optimisation procedures

2.4.1. Rationale and description

The optimisation procedure was developed to establish what local activation control patterns of embedded active elements (embedded as an array of reinforcement bars within a soft polymeric structure made of one or several layers) are necessary to deform a surface to match a specified 3D shape.
Based on the choice of material properties and geometric characteristics of a multilayer soft surface containing particular arrangements of active contractile/extensible elements (that could be made of electroactive polymers), the user can choose a 3D target shape that is specified by providing an analytical formula but which could equally be obtained by reverse engineering real surfaces using laser or tomographic techniques. The computational platform is then used to run an inverse finite element optimisation study that calculates the localised active deformations required in each reinforcement bar element to produce the targeted shape.

2.4.2. Active elements

For this proof-of-concept project, the real physical phenomena driving the active linear contraction/extension of the reinforcement elements were not important to model. Unidimensional deformations of the reinforcement bar elements were prescribed by specifying a single contraction/extension scalar parameter ϕ so that the stretch of the element was $\Delta\lambda = \lambda_0 + \phi$ with λ_0 being the initial stretch values (equal to 1 in a stress-free configuration). This is based on a simple thermal analogy. Elements were implemented with quadratic shape functions and used Lobatto integration rules [35, 36].
Kinematic coupling procedures were developed to embed bar elements (by projection) into shell elements composing the top surface of the multi-layer structure. A special projection technique was implemented so that any type of reinforcement bar element geometry could be used.

2.4.3. Numerical optimisation

In this project, only linear constraints were considered for the optimisation problem:

- Minimum and maximum elongations of reinforcement bar elements
- Targeted displacement field required to morph the original multi-layer structure into the targeted shape
- Maximum deformation of the mesh of each solid layer.

The objective function and all constraints are linear functions of the optimisation variables (ϕ_i), i being the identifier of each reinforcement bar element). The problem is therefore a **linear programming (LP) problem**. This type of optimisation problem can be efficiently solved using a **simplex method** [40]. Solution procedures for non-linear optimisation problems can be viewed a sequence of linear problems and can be amenable to a LP form. The optimum solution of an LP problem corresponds to one of the basic feasible solutions and thus can be found by examining all basic solutions. However, depending on the particular problem, the number of possible basic solutions can be very large. It is therefore desirable to quickly find the basic feasible solution which minimises the objective function without examining all possibilities.
The basic idea of the simplex method is to start with a basic feasible solution and then seek a neighbouring basic feasible solution which reduce the value of the objective function further [40].

The optimisation procedure is a computational loop exploiting the (consistently derived) analytical gradient and sensitivities of the objective function to the control parameters (activation deformation of the reinforcement bar elements) to execute a series of non-linear finite element analyses. When the target shape is reached the optimisation procedure is stopped.

Remark 3

*The overall robustness of the global procedure was found to be mainly conditioned by the non-linear finite element solving procedure. For this reason a special arc-length solver capable of handling critical points of the equilibrium path such as limit points, bifurcations points, snap-back and snap-though [36].This was detailed in section **2.3**.*

2.5 WP4: Development and implementation of advanced algorithms for tensor visualisation

In this work package, advanced eigenvalue/eigenvector extraction techniques and tensor visualisation algorithms were extended/implemented in ParaView. Tensor field visualisation in solid mechanics has not been fully embraced by researchers and engineers despite the physical insight that such technique could provide to interpret complex multi-dimensional data sets [41, 42]. Typical visualisation techniques for tensor field data consist of scalar surface colour plot of one component of the tensor or one of its eigenvalues. In that case, the directional information contained in the tensor is lost. Typical tensors encountered in continuum mechanics are strain, stress, permeability, diffusion, fabric or damage tensors. Anisotropic behaviour, whether it concerns mechanical properties, residual stress, electrical/magnetic permittivity can be captured and visualised using tensor formalism.

There are different methods to visualise tensors. For example, one can calculate the eigenvalues and associated eigenvectors of the tensor. Each eigenvector defined at each point of space (each integration point or each node in the context of finite element analyses) represents a vector field. Similar to classical flow visualisation techniques, one can integrate and plot lines tangent to the vector field. These lines can be further transformed into tubes (or other axially-symmetric primitives) to enhance the perception of depth (**Figure 4**). Tubes can be scaled by the associated eigenvalues. Another way to represent a tensor field consists in plotting a 2D or 3D icon at each point which accounts for the local characteristics of the tensor field. For example, ellipsoids are widely used to represent symmetric tensors. The principal axes of the ellipsoid are aligned with the three eigenvector directions and scaled according to the corresponding eigenvalues (**Figure 5**).

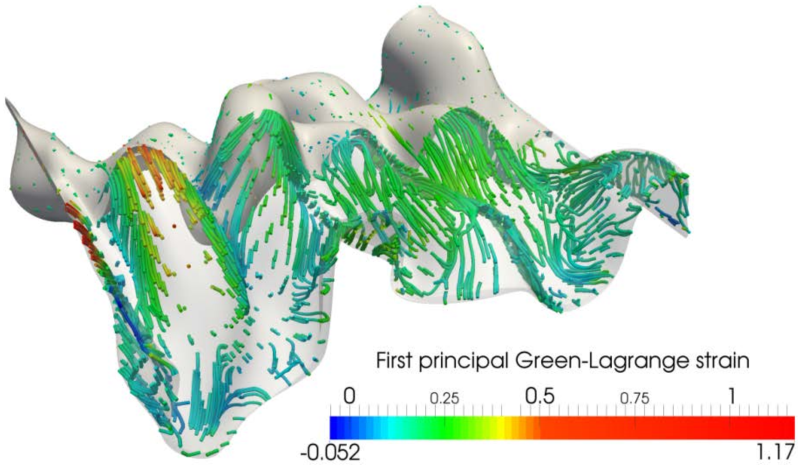


Figure 4. Visualisation of the first principal Green-Lagrange strain vector field (using streamlines) and the associated eigenvalue (first principal strain) in a wrinkled isotropic thin structure after application of an in-plane compression.

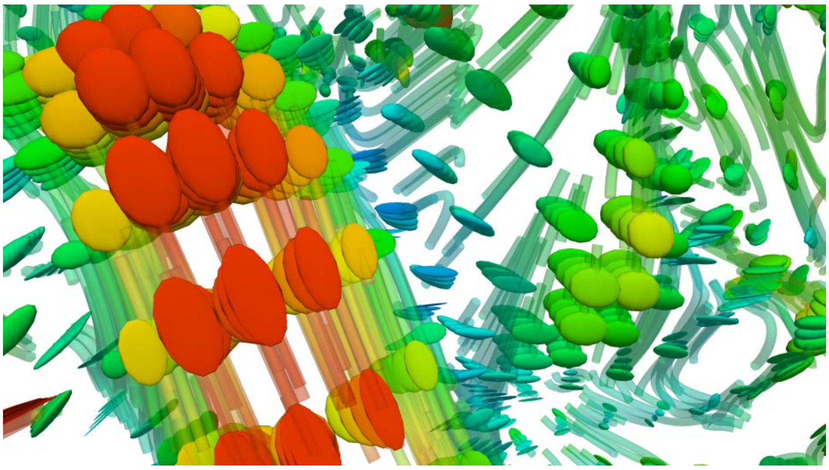


Figure 5. Close-up view of tensor ellipsoids and tubed streamlines representing the three principal Green-Lagrange strain vector fields in the structure presented in **Figure 4**.

For this project, the ability to visualise strain and stress tensors will prove very useful when considering complex arrangements of active elements and/or anisotropic materials and/or structures made of pre-stressed polymer sheets and/or multiphysics materials (electro- and/or magneto-active). Moreover, novel techniques to visualise non-symmetric tensors [43] could be also implemented in the future.

3. Exploitation of the computational platform

The exploitation of the computational platform developed in the first part of this project (see section **2**) **demonstrated the feasibility of the proposed concept for tunable shape-shifting or morphing structures**.

3.1 Computational platform to design a shape-shifting or morphing structure

A typical computational workflow for determining surface tuning parameters can be broken down in the following steps:

1. Build a multi-layer cuboid solid

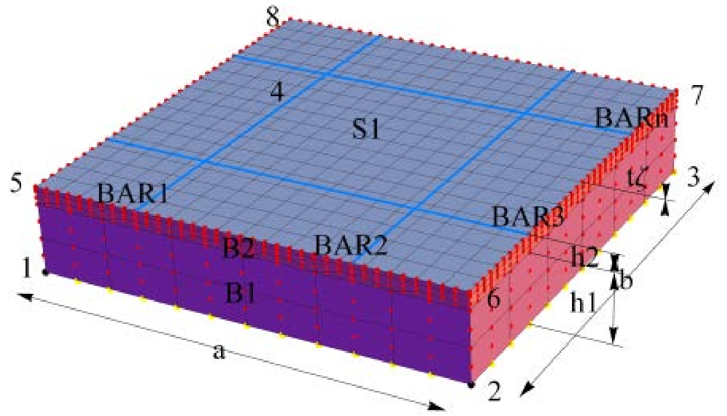


Figure 6. Solid representation of a 3-layer structure of dimensions a×b×(h1+h2+t) reinforced by 4 bar elements (BAR1, BAR2, BAR3 and BAR4)
B1: solid layer 1
B2: solid layer 2
S1: shell layer 1

2. Embed reinforcement bar elements (number of bar reinforcements can be arbitrarily chosen) can be in the shell surface (S1) according to a specific geometric pattern.
3. Assign material properties to each of the layers. In this project, simple isotropic hyperelastic materials (neo-Hookean formulation [44]) were used but other types of materials (e.g. anisotropic hyperelastic/hyperviscoplastic) could be seamlessly used in this framework. For example, in any follow-up study of this project, constitutive models for electroactive polymers could be implemented.
4. Apply periodic boundary conditions to the lateral faces of the cuboid and prevent vertical displacement of the bottom face of the base layer (B1).
5. Specify a target shape for the top surface of the multi-layer structure by providing an analytical equation $z = f(x,y)$.
6. If needed, specify additional constraints (e.g. maximum extension/contraction of reinforcement bar elements).
7. Run the optimisation procedure to determine activation patterns for the reinforcement bar elements that lead to deformations matching the target 3D topography of the surface. The optimisation procedure is a loop exploiting the gradient and sensitivities of the objective function to the control parameters (activation deformation of the reinforcement bar elements) to execute a series of non-linear finite element analyses. When the target shape is reached the optimisation procedure is stopped
8. Obtain the activation patterns for the reinforcement bar elements.

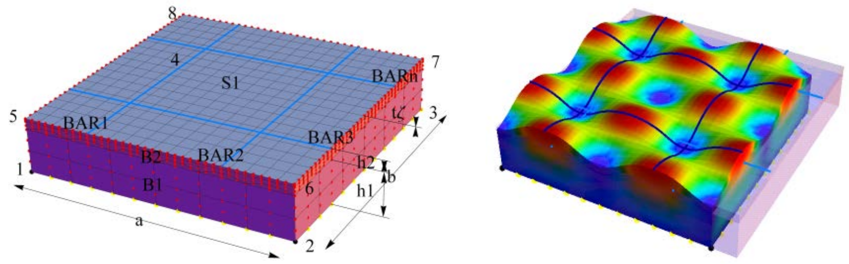
Remark 4

Computational cost: *for the examples considered, the total computational time of the optimisation procedure varied between 20 and 65 minutes. No doubt, this could be accelerated using a larger number of CPUS and/or exploiting the excellent capabilities of modern GPU architecture.*

3.2 Examples of morphed surfaces

In this section examples of morphed surfaces are highlighted. The reference multi-layer structure is the one depicted on **Figure 6**, reproduced for convenience on **Figure 7**. It is important to note that the following characteristics were considered:

- The 3 layers and active elements were made of isotropic hyperelastic materials (neo-Hookean formulation)
- **Solid layer B1**: 10 MPa initial Young's modulus(0.45 Poisson's ratio) / 1.6 mm thickness
- **Solid layer B2**: 50 MPa initial Young's modulus(0.45 Poisson's ratio) / 0.4 mm thickness
- **Shell layer S1**: 500 MPa initial Young's modulus(0.45 Poisson's ratio) / 0.08 mm thickness
- **Active element**: 5000 MPa initial Young's modulus(0.3 Poisson's ratio)



a/ b/

Figure 7. a/ Multi-layer structure reinforced by 4 active bar elements in its reference configuration; b/ Same structure after active deformation of the bar elements. The colour corresponds to the vertical displacement.

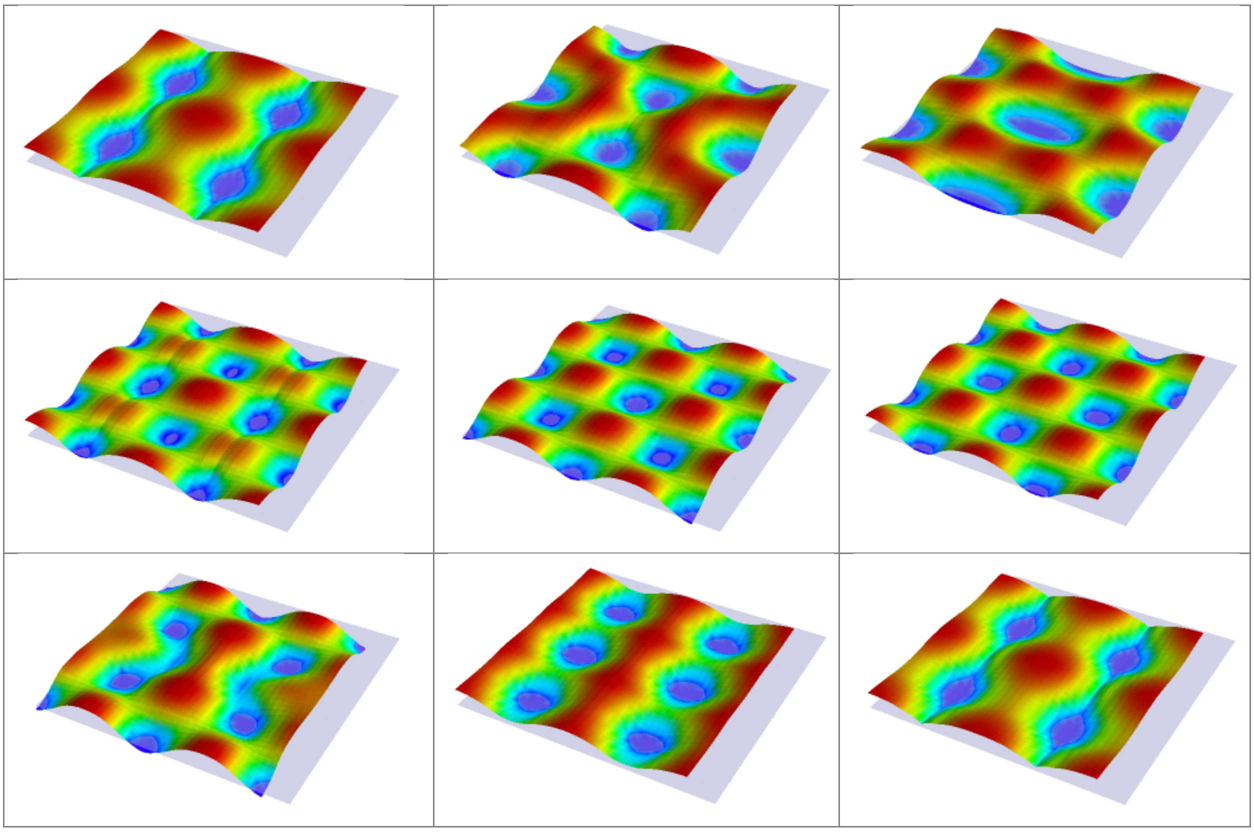


Figure 8. Examples of target surface topographies obtained by optimisation of active deformations of reinforcement bar elements. The initial configuration is that showed in **Figure 7-a**. The grey surface represents the top surface of the multi-layer structure before application of active deformations.

4. Conclusions and opportunities for further research and development

The modelling project supported by EOARD **[FA8655-12-1-2103]** demonstrated that it is possible to three-dimensionally deform the surface of a multi-layer soft structure to match a given target surface topography by inducing controlled localised buckling. This controlled buckling is achieved by active contraction and/or extension of active reinforcement bar elements embedded in the top surface layer of a given polymeric material. Combining several layers (with distinct properties) in the same structure tremendously increase the number of potential shape configurations. The computational platform that was developed is general and can accommodate various types of active materials acting as controllable actuators. It is only a question of implementing the appropriate multiphysics constitutive models (e.g. electro-mechanics) to virtually design active surfaces made of commercially available or purpose-built electroactive polymers.

4.1 Some thoughts

At this preliminary stage, we have only scratched the surface of what is possible. From section **3.2** (**Figure 8**), one can appreciate the variety of morphed surfaces that can be obtained with a very simple arrangement of 4 reinforcement bar elements.
It is noteworthy that **what differentiates the obtained morphed surfaces is solely the pattern of active deformations** in the reinforcement bar elements that was determined through an integrated optimisation procedure.
It is therefore legitimate to speculate that variations in individual layer thickness and mechanical properties (e.g. increased/reduced stiffness or introduction of anisotropic materials) could lead to an even greater level of control for the type of active surfaces considered in this project.
Also, during the iterative optimisation procedure, before numerical convergence to the target shape, intermediate 3D shapes are obtained. These shapes could be saved and automatically analysed (using for example Fourier transform or multiphysics-finite element simulations) to extract potential interesting physical properties (e.g. acoustic, optical, mechanical) that were not anticipated.
The computational platform can be used to study morphing structures operating from the nanoscopic to the macroscopic scale. Naturally, if considering small scales, surface physics phenomena such as van der Waals forces [45] would have to be included. Because of the modular nature of the computational platform it would be straightforward to implement these types of force using dedicated field variables besides displacement.

4.2 Applications

As already highlighted, reconfigurable active surfaces have a huge potential in a military (and also civil) context. The proposed innovation could not only accelerate research and development in metamaterials and active surfaces but could also improve the performance of existing/future systems by integrating reconfigurable multi-functionality. Selected potential applications (*all with the added benefit of being reconfigurable*) of the proposed research are:

- smart surfaces that can change their radar, acoustic, thermal and/or optical signatures (camouflage)
- adjustable inter-locking mechanisms for under-sea/space structures;
- directional textured surfaces;
- directional friction surfaces to enhance manoeuvrability of aerospace and under-sea vehicles;
- patterned surfaces for drag reduction;
- anti-biofouling patterned surfaces for over- and under-sea vehicles/structures
- active cooling
- context-sensitive reconfigurable control surfaces (e.g. keyboards switching from one language to another).

4.3 Future work

This project mainly focused on developing a numerical toolbox and environment to model reconfigurable active structures. Exploitation of the modelling platform has been initiated but a vast amount of exploratory work remains to be done. Large scale parametric analyses combining active deformations with variations in geometrical and mechanical properties and also arrangement of active reinforcement elements (e.g. circular, concentric, butterfly wings, spider web) should be conducted. Multiobjective design optimisation could also be conducted to design surface topographies fulfilling particular physical properties (e.g. wave refraction).

In this project the physics triggering active deformations within the reinforcement bar elements was not accounted for. It would therefore make sense that, in a sequel to this project, realistic physical mechanisms for active deformations be implemented in the computational platform. One could envisage the experimental characterisation of various formulations of electroactive polymers to guide the development of a physically-sound multiphysics constitutive model of electroelasticity. The active deformations of simple samples made of these polymers could be tested and used to validate the constitutive modelling aspects. After validation of the electroactive characteristics of the polymers, one could develop a physical multi-layer prototype structure and use the computational platform to establish active control patterns necessary to obtain target shapes. The results of the numerical optimisation procedure could then be used to control the physical prototype and, hopefully, validate the design platform.

5. References

[1] GENZER, J. & GROENEWOLD, J. 2006. Soft matter with hard skin: From skin wrinkles to templating and material characterization. *Soft Matter,* 2 (4), 310-323.

[2] SCHWEIKART, A. & FERY, A. 2009. Controlled wrinkling as a novel method for the fabrication of patterned surfaces. *Microchimica Acta,* 165 (3-4), 249-263.

[3] BHUSHAN, B. 2009. Biomimetics: lessons from nature - an overview. *Philosophical Transactions of the Royal Society A-Mathematical Physical and Engineering Sciences,* 367 (1893), 1445-1486.

[4] SHOHET, A. J., BADDELEY, R. J., ANDERSON, J. C., KELMAN, E. J. & OSORIO, D. 2006. Cuttlefish responses to visual orientation of substrates, water flow and a model of motion camouflage. *Journal of Experimental Biology,* 209 (23), 4717-4723.

[5] HANLON, R. 2007. Cephalopod dynamic camouflage. *Current Biology,* 17 (11), R400-R404.

[6] ALLEN, J. J., MATHGER, L. M., BARBOSA, A. & HANLON, R. T. 2009. Cuttlefish use visual cues to control three-dimensional skin papillae for camouflage. *Journal of Comparative Physiology a-Neuroethology Sensory Neural and Behavioral Physiology,* 195 (6), 547-555.

[7] RICHARDS, A. W. & ODEGARD, G. M. 2010. Constitutive Modeling of Electrostrictive Polymers Using a Hyperelasticity-Based Approach. *Journal of Applied Mechanics,* 77 (1), 014502.

[8] DORFMANN, A. & OGDEN, R. W. 2004. Nonlinear magnetoelastic deformations. *Quaterly Journal of Applied Mathematics,* 57 (4), 599-622.

[9] HUGHES, T. J. R., COTTRELL, J. A. & BAZILEVS, Y. 2005. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering,* 194 (39-41), 4135-4195.

[10] GROENEWOLD, J. 2001. Wrinkling of plates coupled with soft elastic media. *Physica A,* 298 (1-2), 32-45.

[11] CERDA, E., RAVI-CHANDAR, K. & MAHADEVAN, L. 2002. Thin films - Wrinkling of an elastic sheet under tension. *Nature,* 419 (6907), 579-580.

[12] CERDA, E. & MAHADEVAN, L. 2003. Geometry and physics of wrinkling. *Physical Review Letters,* 90 (7), 074302-1:074302-4.

[13] GENZER, J. & EFIMENKO, K. 2006. Recent developments in superhydrophobic surfaces and their relevance to marine fouling: a review. *Biofouling,* 22 (5), 339-360.

[14] LANDAU, L. & LIFSHITZ, E. M. 1986. *Theory of elasticity,* New York, Pergamon.

[15] CIRAK, F. & ORTIZ, M. 2001. Fully C^1-conforming subdivision elements for finite deformation thin-shell analysis. *International Journal for Numerical Methods in Engineering,* 51 (7), 813-833.

[16] CIRAK, F., ORTIZ, M. & SCHRÖDER, P. 2000. Subdivision surfaces: a new paradigm for thin-shell finite-element analysis. *International Journal for Numerical Methods in Engineering,* 47 (12), 2039-2072.

[17] FORRESTER, A., SOBESTER, A. & KEANE, A. 2008. *Engineering design via surrogate modelling: a practical guide* Chichester, UK, John Wiley & Sons.

[18] KEANE, A. J. & NAIR, P. B. 2005. *Computational approaches for aerospace design: the pursuit of excellence*, John Wiley & Sons.

[19] KORELC, J. 2002. Multi-language and multi-environment generation of nonlinear finite element codes. *Engineering with Computers,* 18 (4), 312-327.
[20] COTTRELL, J. A., HUGHES, T. J. R. & BAZILEVS, Y. 2009. *Isogeometric analysis: toward integration of CAD and FEA,* Oxford, Wiley.
[21] PIEGL, L. & TILLER, W. 1997. *The NURBS Book,* New York, Springer-Verlag.
[22] BAZILEVS, Y. & HUGHES, T. J. R. 2008. NURBS-based isogeometric analysis for the computation of flows about rotating components. *Computational Mechanics,* 43 (1), 143-150.
[23] COTTRELL, J. A., REALI, A., BAZILEVS, Y. & HUGHES, T. J. R. 2006. Isogeometric analysis of structural vibrations. *Computer Methods in Applied Mechanics and Engineering,* 195 (41-43), 5257-5296.
[24] BAZILEVS, Y., CALO, V. M., HUGHES, T. J. R. & ZHANG, Y. 2008. Isogeometric fluid-structure interaction: theory, algorithms, and computations. *Computational Mechanics,* 43 (1), 3-37.
[25] BAZILEVS, Y., CALO, V. M., ZHANG, Y. & HUGHES, T. J. R. 2006. Isogeometric fluid-structure interaction analysis with applications to arterial blood flow. *Computational Mechanics,* 38 (4-5), 310-322.
[26] HUGHES, T. J. R., REALI, A. & SANGALLI, G. 2008. Duality and unified analysis of discrete approximations in structural dynamics and wave propagation: Comparison of p-method finite elements with k-method NURBS. *Computer Methods in Applied Mechanics and Engineering,* 197 (49-50), 4104-4124.
[27] BENSON, D. J., BAZILEVS, Y., HSU, M. C. & HUGHES, T. J. R. 2010. Isogeometric shell analysis: The Reissner-Mindlin shell. *Computer Methods in Applied Mechanics and Engineering,* 199 (5-8), 276-289.
[28] BENSON, D. J., BAZILEVS, Y., HSU, M. C. & HUGHES, T. J. R. 2011. A large deformation, rotation-free, isogeometric shell. *Computer Methods in Applied Mechanics and Engineering,* 200 (13-16), 1367-1378.
[29] WALL, W. A., FRENZEL, M. A. & CYRON, C. 2008. Isogeometric structural shape optimization. *Computer Methods in Applied Mechanics and Engineering,* 197 (33-40), 2976-2988.
[30] HASSANI, B., TAVAKKOLI, S. M. & MOGHADAM, N. Z. 2011. Application of isogeometric analysis in structural shape optimization. *Scientia Iranica,* 18 (4), 846-852.
[31] NGUYEN-THANH, N., KIENDL, J., NGUYEN-XUAN, H., WUCHNER, R., BLETZINGER, K. U., BAZILEVS, Y. & RABCZUK, T. 2011. Rotation free isogeometric thin shell analysis using PHT-splines. *Computer Methods in Applied Mechanics and Engineering,* 200 (47-48), 3410-3424.
[32] KIENDL, J., BLETZINGER, K. U., LINHARD, J. & WUCHNER, R. 2009. Isogeometric shell analysis with Kirchhoff-Love elements. *Computer Methods in Applied Mechanics and Engineering,* 198 (49-52), 3902-3914.
[33] KIENDL, J., BAZILEVS, Y., HSU, M. C., WUCHNER, R. & BLETZINGER, K. U. 2010. The bending strip method for isogeometric analysis of Kirchhoff-Love shell structures comprised of multiple patches. *Computer Methods in Applied Mechanics and Engineering,* 199 (37-40), 2403-2416.
[34] BORDEN, M. J., SCOTT, M. A., EVANS, J. A. & HUGHES, T. J. R. 2011. Isogeometric finite element data structures based on Bézier extraction of NURBS. *International Journal for Numerical Methods in Engineering,* 87 (1-5), 15-47.
[35] BELYTSCHKO, T., LIU, W. K. & MORAN, B. 2000. *Nonlinear Finite Elements for Continua and Structures,* Oxford, Wiley.
[36] WRIGGERS, P. 2008. *Nonlinear Finite Elment Methods,* Berlin - Heidelberg, Springer-Verlag.
[37] CRISFIELD, M. A. 1991. *Non-linear finite element analysis of solids and structures - Volume 1,* Chichester ; New York, Wiley.
[38] DE SOUZA NETO, E. A. & FENG, Y. T. 1999. On the determination of the path direction for arc-length methods in the presence of bifurcations and `snap-backs'. *Computer Methods in Applied Mechanics and Engineering,* 179 (1–2), 81-89.
[39] SCHWEIZERHOF, K. H. & WRIGGERS, P. 1986. Consistent linearization for path following methods in nonlinear fe analysis. *Computer Methods in Applied Mechanics and Engineering,* 59 (3), 261-279.
[40] BHATTI, A. 2000. *Practical optimization methods: with Mathematica applications,* New York, Springer-Verlag.
[41] KRATZ, A., MEYER, B. & HOTZ, I. 2010. A visual approach to analysis of stress tensor fields. *ZIB Report,* August 2010, 1-14.
[42] HOTZ, I., FENG, L., HAGEN, H., HAMANN, B., JOY, K. & JEREMIC, B. 2004. Physically based methods fro tensor field visualization. *IEE Visualization*, 123-130.
[43] KRATZ, A., AUER, C., STOMMEL, M. & HOTZ, I. 2013. Visualization and analysis of second-order tensors: moving beyond the symmetric poistive-definite case. *Computer Graphics Forum,* 32, 49-74.
[44] HOLZAPFEL, G. A. 2000. *Nonlinear Solid Mechanics. A Continuum Approach for Engineering,* Chichester, UK, John Wiley & Sons.
[45] ISRAELACHVILI, J. N. 2011. *Intermolecular and surface forces,* Burlington, MA, USA, Academic Press.

6. Appendix A – Journal paper submitted

Paper submitted to the journal **Computer Methods in Applied Mechanics and Engineering**:

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Abstract: NURBS-based isogeometric analysis was first extended to thin shell/membrane structures which allows for finite membrane stretching as well as large deflection and bending strain. The assumed non-linear kinematics employs the Kirchhoff-Love shell theory to describe the mechanical behaviour of thin to ultra-thin structures. The displacement fields are interpolated from the displacements of control points only, and no rotational degrees of freedom are used at control points. Due to the high order Ck (k ≥ 1) continuity of NURBS shape functions the Kirchhoff-Love theory can be seamlessly implemented. An explicit time integration scheme is used to compute the transient response of membrane structures to time-domain excitations, and a dynamic relaxation method is employed to obtain steady-state solutions. The versatility and good performance of the present formulation is demonstrated with the aid of a number of test cases, including a square membrane strip under static pressure, the inflation of a spherical shell under internal pressure, the inflation of a square airbag and the inflation of a rubber balloon. The mechanical contribution of the bending stiffness is also evaluated.